Enrollment No: ____________________ **Exam Seat No:** ____________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Advanced Calculus

Subject Code: 4SC03ADC1/4SC03MTC1 **Branch: B.Sc. (Mathematics, Physics)**

Semester: 3 **Date: 29/11/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) If $u = \tan^{-1}(x+y)$ then $u_x - u_y =$ ______. (02)

b) If $z = f(u,v)$, $u = g(x,y)$ and $v = h(x,y)$ then $\frac{\partial z}{\partial x} =$ ________. (02)

c) If $f(x,y,z) = \frac{x^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{x^2}$ then find the value of $xf_x + yf_y + zf_z$. (02)

d) If u & v are functions of x & y then $JJ' =$ ______. (01)

e) Write the relation between beta and gamma function. (01)

f) Write Duplication formula (01)

g) Find: $\Gamma(3.5)$ (01)

h) Evaluate: $B\left(\frac{1}{4}, \frac{3}{4}\right)$ (02)

i) Define: Concave upwards, Inflection point (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $z(x+y) = x^2 + y^2$ then show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. (05)

b) Discuss whether the function $f(x,y) = \begin{cases} \frac{x^3y^3}{x^3+y^3}; & (x,y) \neq (0,0) \\ 0 \quad ; & (x,y) = (0,0) \end{cases}$ is continuous at origin. (05)

c) If $y = f(x+at) + g(x-at)$ then prove that $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$. (04)

Q-3 Attempt all questions **(14)**

a) Find the extreme value of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$. (05)

b) Evaluate: $\int_0^1 (x\log x)^4\,dx$ (05)

c) If the function $z=f(x,y)$ possesses the first order derivatives in the domain D and $x=g(t), y=h(t)$ also possesses its first order partial derivative then show that $\frac{dz}{dt}=\frac{\delta z}{\delta x}\frac{dx}{dt}+\frac{\delta z}{\delta y}\frac{dy}{dt}$. (04)

Q-4 Attempt all questions **(14)**

a) State and prove Euler's theorem for homogeneous function. (05)

b) If $u=x^y$ then show that $\frac{\partial^3 u}{\partial x^2 \partial y}=\frac{\partial^3 u}{\partial x \partial y \partial x}$. (05)

c) If $u=x^2+y^2+z^2$ and $x=e^{2t},\ y=e^{2t}\cos 3t,\ z=e^{2t}\sin 3t$ then find $\frac{du}{dt}$. (04)

Q-5 Attempt all questions **(14)**

a) State and prove Taylor's series for function of two variables. (07)

b) Evaluate: $\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx$ (04)

c) Find the maximum value of $V(x,y,z)=xyz$ subjected to the constraint $2x+2y+z=108$. (03)

Q-6 Attempt all questions **(14)**

a) If $u=\tan^{-1}(x^2+y^2)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cdot\cos 3u$. (05)

b) If $x=r\cos\theta,\ y=r\sin\theta$ then prove that $\frac{\partial(x,y)}{\partial(r,\theta)}\cdot\frac{\partial(r,\theta)}{\partial(x,y)}=1$. (05)

c) Prove that $B(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cdot\cos^{2n-1}\theta\,d\theta$. (04)

Q-7 Attempt all questions **(14)**

a) If $\theta=t^n e^{\frac{-r^2}{4t}}$ and $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$ then find the value of n. (05)

b) Find the asymptote of the following: (05)

i) $x^3+y^3-3xy=0$

ii) $4x^2+9y^2=16x^2y^2$

c) Prove that $\Gamma(n) = 2\int_0^{\infty} e^{-x^2} x^{2n-1}\, dx$. (04)

Q-8 Attempt all questions **(14)**

a) Evaluate: $\int_0^{2a} x\sqrt{2ax - x^2}\, dx$ (05)

b) Find the point of inflection of the following: (05)

i) $f(x) = (x-2)^3 (x-3)^2$

ii) $f(x) = x^5 - 5x^4 + 5x^3 - 1$

c) Expand $e^x \log(1+y)$ in powers of x and y up to third degree terms. (04)